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Yield Benefit of Corn Event MON 863

Data Requirement

None

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Executive Summary

Data from field experiments are used to estimate the yield benefit of corn hybrids containing event MON 863 relative to nontransgenic corn hybrids without corn rootworm control and with a soil insecticide for corn rootworm control. Over typical ranges for corn rootworm population pressure, event MON 863 provides a yield benefit of 9-28% relative to no control and of 1.5-4.5% relative to control with a soil insecticide. For a reasonable range of prices and yields, the value of the event MON 863 yield benefit is \$25-\$75/ac relative to no control and \$4-\$12/ac relative to control with a soil insecticide, depending on corn rootworm pressure.

Because of the low correlation between yield loss and the root rating difference, a common empirical finding when estimating yield loss with root ratings, the 95% confidence intervals around these averages are quite wide. Though on average, event MON 863 has substantial value, the wide confidence intervals imply that farmers will see a wide variety of actual performance levels in their fields. This uncertainty in the realized yield benefit is not due to any property of event MON 863, but rather due to the inherent randomness in the numerous environmental and agronomic factors determining a corn plant's yield and yield response to corn rootworm larval feeding damage.

Introduction

This report estimates the yield benefit of corn hybrids containing event MON 863, the new transgenic corn event developed by Monsanto that provides control of western and northern corn rootworm larvae. The yield benefit of event MON 863 is estimated relative to no corn rootworm control and relative to control with a soil insecticide. This analysis is not a full accounting of the benefits of hybrids containing event MON 863, only the yield benefit. Event MON 863 provides other benefits to farmers and to society that this analysis does not include. For example, most farmers adopting hybrids containing event MON 863 will reduce insecticide use, but the human health and environmental benefits to farmers and society of this decrease are not included.

As is typical early in the product development process for transgenic corn events, initial field evaluations were conducted with the transgenic event in non-elite hybrids. As a result, no yield data were collected, since such data would not be indicative of actual yield performance of the event in elite hybrids. Rather, field performance was measured by the standard 1-6 root rating scale of Hills and Peters (1971). Field data from Gray and Steffey (1998) are used to estimate proportional yield loss as a function of the root rating difference. Applying this model to data from field trials with hybrids containing event MON 863 provides an estimate of the yield benefit of event MON 863 relative to no corn rootworm control and relative to control with a soil insecticide.

Data used for Estimations

Gray and Steffey (1998) Data

Three years (1994-1996) of data from experiments conducted by Gray and Steffey (1998) in 2 locations in Illinois (near Urbana and DeKalb) were used for this analysis.

Whole plot treatments were 12 commonly grown nontransgenic maize hybrids. Sub-plot treatments were 2 rows treated with terbufos (Counter 15G) and 2 untreated rows. Depending on the year and location, 8-10 replicates for each hybrid were planted. Collected data included machine-harvested yield and the average root rating for five plants on the 1-6 scale of Hills and Peters (1971). Only data with treated and untreated yields and root ratings for both sub-plots were used, so that both the root rating difference and proportional yield loss could be calculated. The final result was 621 observations of yield (bu/ac) and average root rating for the paired untreated and soil insecticide treated sub-plots. For a more complete description, see Gray and Steffey (1998).

Tables 1 and 2 summarize the data. The root rating difference ΔR is calculated as

(1) $$\Delta R = \left| R_N - R_S \right|.$$

R_N is the average root rating for the untreated sub-plot and R_S is the average root rating for the soil insecticide sub-plot. When $R_N > R_S$, proportional yield loss λ is calculated as

(2) $$\lambda = (Y_S - Y_N) / Y_S,$$

where Y_N is the yield for the untreated subplot and Y_S is the yield for the soil insecticide sub-plot. So that proportional yield loss is relative to yield from the plot with the lowest root rating, for the nine instances when $R_S > R_N$, proportional yield loss is calculated as

(3) $$\lambda = (Y_N - Y_S) / Y_N.$$

Gray and Steffey (1998) report statistically significant differences in yields and root ratings for some site-years. However, a model with site-year effects is not appropriate for the analysis here, since the type of site-year that will occur is in general unknown. As a result, the data are pooled to estimate a function that averages across years and locations to determine average yield loss given the root rating difference.

Monsanto Event MON 863 Efficacy Data

Data from efficacy experiments conducted in 1999 and 2000 in several locations were used to estimate the impact of event MON 863 on the root rating. Monsanto (2000) and Pilcher (2001) describe the experiments and report the average root rating for each treatment at each location. Plots were artificially infested or had a trap crop planted the previous season to ensure high western corn rootworm larval populations. Table 3 reports all the data used for this analysis.

Model

Proportional Yield Loss as a Function of the Root Rating Difference

Several factors contribute to any observed yield difference between an untreated plot and a treated plot. Soil characteristics vary, tillage and nutrient applications are not uniform, yields are measured with error, and experimental treatments are applied with error. As a result, yield variability due to soil heterogeneity, non-uniformity of tillage, nutrient and pesticide applications, and yield measurement error is confounded with yield variability due to the treatment. A statistical model is needed that separates the treatment effect on yield from the effect of the other "noise" factors.

The composed error model of Mitchell, Gray and Steffey (Appendix A) separates proportional yield loss into (1) a mean-zero normal error to capture yield variability due to soil heterogeneity, non-uniformity of agronomic practices, measurement errors, and similar factors and (2) a strictly positive error to capture yield loss due to corn rootworm damage. Specifically, the model assumes proportional yield loss λ is composed of two independent errors—a normal (Gaussian) error ε and a strictly positive error δ:

(4) $$\lambda = 1 - \exp(-(\delta + \varepsilon)).$$

The normal error ε has a zero mean and variance σ^2, while δ has an exponential distribution with mean θ. When the untreated yield is zero, $\lambda = 1$ (a 100% loss) by equation (2). When the yield with soil insecticide is zero, $\lambda = -\infty$ by equation (2). Equation (4) imposes these same limits on λ.

Maximum likelihood is used to estimate the parameters of the probability density function for λ, conditional on the root rating difference. The mean of this conditional distribution is the average proportional yield loss as a function of the root rating difference. The same six conditional models estimated by Mitchell, Gray and Steffey (Appendix A) were estimated with these data. Table 4 reports the results. The analysis here uses the linear model because it provides the best fit as measured by the adjusted R^2 and the root mean square error (RMSE), and is parsimonious in terms of the number of parameters. Figure 1 illustrates the model fit.

None of the models provides a high adjusted R^2 or a low RMSE. As Figure 1 shows, proportional yield loss varies substantially when the root rating differences is constant. As a result, a low correlation between proportional yield loss and the root rating difference exists ($\rho = 0.44$) and a low adjusted R^2 occurs. The positive relationship between root rating and yield loss is well established in the literature (Turpin et al. 1972, Stamm et al. 1985, Sutter et al. 1990, Davis 1994, Urias-Lopez and Meinke 2001), though empirically a low correlation is common. Even under equal agronomic treatment, corn yields are highly variable within a field. The low correlation between root ratings and yield loss occurs because numerous environmental and agronomic factors together determine a corn plant's yield and yield response to root damage.

The purged form of the conditional model is appropriate for the analysis here, since the focus is on the impact of corn rootworm control on yield, not the effect of soil heterogeneity and similar factors. As such, the probability density function for λ is

(5) $$h(\lambda) = \left((1-\lambda)^{\frac{1-\theta}{\theta}} \right) / \theta$$

for $0 \le \lambda \le 1$, and 0 otherwise, and the cumulative distribution function is

(6) $$H(\lambda) = 1 - (1-\lambda)^{1/\theta},$$

where $\theta = \alpha\Delta R / (1 - \alpha\Delta R)$. Mean loss is $\mu_\lambda = \alpha\Delta R$. Using the estimated α reported in Table 4 for the linear model, average proportional loss given the root rating difference is

(7) $$\mu_\lambda = 0.125\Delta R$$

and $\theta = 0.125\Delta R / (1 - 0.125\Delta R)$. Inverting the cumulative distribution function gives the lower and upper limits of the 95% confidence interval for the purged model:

(8) $$\lambda_{lower} = 1 - (1 - 0.025)^{\theta},$$

(9) $$\lambda_{upper} = 1 - (1 - 0.975)^{\theta}.$$

On average, over many years and across many locations, Figure 1 shows that the average yield benefit can be quite substantial, depending on the corn rootworm pressure as measured by the untreated root rating. However, Figure 1 also shows that, though on average the yield benefit is substantial, the actual yield benefit realized by a farmer in a particular field for a particular year can easily vary between 0% to almost 100%. As a result, at an aggregate level, such as at a county, state or regional level, soil insecticides on average save a substantial amount of yield, but for an individual farmer, the realized benefit on a specific field during any particular year is quite variable. This uncertainty in the realized benefit results from the inherent uncertainty in the environmental and agronomic factors that determine a corn plant's yield and yield response to root damage.

Soil Insecticide Root Rating as a Function of the Untreated Root Rating

The root rating data from Gray and Steffey (1998) are used to estimate the average root rating for corn treated with a soil insecticide as a function of the untreated root rating, i.e. the root rating for corn receiving no corn rootworm control. Because the root rating is strictly between 1 and 6, ordinary least squares regression is inappropriate, since the assumed normal error ranges between $-\infty$ and $+\infty$. The beta distribution has lower and upper limits like the root rating, and its flexibility make it a good choice for estimating a conditional root rating distribution via maximum likelihood.

Observed root ratings are first rescaled to range between 0 and 1 (the lower and upper limits of the standard beta density) by the following transformation:

(10) $$\tilde{R} = (R-1)/(6-1),$$

where R is any root rating on the 1 to 6 scale and $\tilde{R}$ is the re-scaled root rating between 0 and 1. The standard beta probability density function with parameters ν and γ has mean $\nu/(\nu+\gamma)$ and standard deviation $\sqrt{\nu\gamma/[(\nu+\gamma)^2(\nu+\gamma+1)]}$ (Evans et al. 1993).

This analysis assumes the conditional probability density function for the rescaled soil insecticide root rating $\tilde{R}_S$ is beta with linear mean $\beta_S\tilde{R}_N$ and constant standard deviation σ_S. Equating these to the mean and standard deviation formulas for the standard beta density gives two simultaneous equations that can be solved to obtain $\nu = \frac{(\beta_S\tilde{R}_N)^2(1-\beta_S\tilde{R}_N)-\beta_S\tilde{R}_N\sigma_S^2}{\sigma_S^2}$ and $\gamma = \frac{(\beta_S\tilde{R}_N)(1-\beta_S\tilde{R}_N)^2-(1-\beta_S\tilde{R}_N)\sigma_S^2}{\sigma_S^2}$ (Evans et al. 1993). Substituting these into the beta probability density function gives the density function of the rescaled root rating $\tilde{R}_S$ conditional on the observed $\tilde{R}_N$, which allows maximum likelihood estimation of the parameters β_S and σ_S as reported in Table 5.

Using estimated parameters and transforming back to the 1 to 6 scale, the average root rating for soil insecticide corn treated as a function of the untreated root rating is

(11) $$R_S = 1 + \beta_S (R_N - 1).$$

Using this conditional mean for the model prediction gives a RMSE of 0.392. Figure 2 illustrates the model fit.

Event MON 863 Root Rating as a Function of the Untreated Root Rating

Repeating the process used to obtain equation (11), but with the efficacy data in Table 3, gives the event MON 863 root rating as a function of the untreated root rating. Table 5 reports maximum likelihood parameter estimates for β_M and σ_M. Given these,

(12) $$R_M = 1 + \beta_M (R_N - 1),$$

where R_M is the average root rating for event MON 863. Using this conditional mean for the model prediction gives a RMSE of 0.390. Figure 3 illustrates the model fit.

Untreated Root Ratings

Equations (11) and (12) indicate that the untreated root rating is needed to obtain ΔR and then to use equation (7) to determine the average proportional yield loss. Unfortunately, few data concerning the untreated root rating for naturally infested corn are available. Most field experiments evaluating soil insecticides or other control methods plant a trap crop the previous season or use artificial infestation to ensure a high corn rootworm larval population. However, published literature provides some indication of untreated root ratings with and without trap crops.

Table 6 summarizes average root ratings for untreated plots for the Gray and Steffey (1998) experiments, which used a trap crop. The overall average of the averages is 4.11, but the averages range between 2.84 and 5.19. Table 7 summarizes results from soil insecticide trials conducted by Iowa State University Department of Entomology (1998, 1999), which used a trap crop. The overall average of the average root rating for untreated plots across all locations and years is 4.1, but the averages range between 2.2 and 5.2. Table 8, adapted from Table 1 in O'Neil et al. (2001), reports annual average root ratings for several untreated first-year fields in seven counties in east central Illinois for 1996-1999. The average of the averages is 2.68 and indicates the typical pressure from rotation resistant western corn rootworm laying eggs in soybeans.

Because of the limited data concerning typical untreated root ratings, the analysis here examines the yield benefit of event MON 863 over the full range of untreated root ratings, using the untreated root rating as an index of corn rootworm pressure. The trap crop data indicate that high corn rootworm pressure in these areas implies an untreated root rating of about 4.1. Data from O'Neil et al. (2000) indicate that for rotation resistant corn rootworm, the untreated root rating averages about 2.7. As a result, this analysis uses an untreated root rating of 2 for low corn rootworm pressure, 3 for mode rate pressure, and 4 for high pressure. However, equations and plots are provided for determining the yield benefit for any untreated root rating.

Model Summary

Equation (7) determines average proportional yield loss as a function of the root rating difference. Equations (11) and (12) determine the root rating for soil insecticide treated corn and event MON 863 corn as a function of the untreated root rating. Substituting in estimated parameters, equations (11) and (12) can be used to determine the difference in the various root ratings as a function of the untreated root rating:

(13) $$\Delta R_{N,S} = R_N - R_S = (1-\beta_S)(R_N - 1) = 0.641(R_N - 1)$$

(14) $$\Delta R_{N,M} = R_N - R_M = (1-\beta_M)(R_N - 1) = 0.754(R_N - 1)$$

(15) $$\Delta R_{S,M} = R_S - R_M = (\beta_S - \beta_M)(R_N - 1) = 0.113(R_N - 1).$$

Combining these equations with equation (7) gives the following equations:

(16) $$\lambda_{N,S} = 0.125\Delta R_{N,S} = 0.080(R_N - 1)$$

(17) $$\lambda_{N,M} = 0.125\Delta R_{N,M} = 0.094(R_N - 1)$$

(18) $$\lambda_{S,M} = 0.125\Delta R_{S,M} = 0.014(R_N - 1).$$

Equations (16)-(18) determine the average yield benefit of corn rootworm control (measured as the proportion of yield saved) as a function of the corn rootworm pressure (measured by the untreated root rating). Equation (16) determines the average yield benefit of soil insecticide relative to no corn rootworm control, equation (17) determines the average yield benefit of event MON 863 relative to no corn rootworm control, and equation (18) determines the average yield benefit of event MON 863 relative to control with soil insecticide.

Equations (8) and (9) combined with equations (13)-(15) give the lower and upper limits of the 95% confidence interval. Because field plot data are used to estimate the yield benefit, this confidence interval is for the yield benefit at the individual field level, not at an aggregate level such as for a county, a state, or a region. The confidence interval at these more aggregated levels would be narrower because the yield benefit is averaged over a wider area.

Results

Yield Benefit of Event MON 863 Relative to No Corn Rootworm Control

Figure 4 and Table 9 summarize the average yield benefit for corn hybrids containing event MON 863, measured as the proportion of yield saved, relative to corn without rootworm control. The confidence interval indicates the large level of uncertainty concerning the realized yield benefit on a specific field during a particular season. This uncertainty does not result from any property of event MON 863, but rather is due to the inherent randomness in the numerous environmental and agronomic factors determining a corn plant's yield and yield response to corn rootworm larval feeding damage. At more aggregated levels such as the county, state or region, the average benefit would remain the same, but the confidence interval would be narrower because the area averaged over is larger.

The average yield benefit of event MON 863 relative to no corn rootworm control is quite substantial. At an untreated root rating of 3, the model predicts an average yield benefit of almost 19%. However, farmers are likely to see a wide variety of actual performance levels, since the confidence interval ranges approximately 0%-60%. As a result, though the average yield benefit is substantial, farmers are likely to discount the average yield benefit to adjust for this uncertainty in the yield benefit actually realized. Tables 9 and 10 indicate that the average yield benefit of event MON 863 relative to no corn rootworm control is similar in magnitude to the yield benefit for soil insecticide, and both have comparable confidence intervals. As a result, farmers will likely discount the average yield benefit of event MON 863 for this uncertainty at a level similar to what they currently discount the average yield benefit of a soil insecticide for the uncertainty in the yield benefit that it provides.

Yield Benefit of Event MON 863 Relative to Control with a Soil Insecticide

Figure 5 and Table 11 summarize the yield benefit for a hybrid containing event MON 863, measured as the proportion of yield saved, relative to using a soil insecticide for corn rootworm control. Again, the confidence interval indicates the uncertainty around this average yield benefit. Because the root rating difference between event MON 863 and a soil insecticide is smaller, the confidence interval is narrower. With an untreated root rating of 3, the average yield benefit is 2.8%, though likely to range between 0% and 10%.

Monetary Value of the Event MON 863 Yield Benefit

Because the yield benefit for a hybrid containing event MON 863 is given in terms of the proportion of yield saved, converting this benefit to a monetary value requires assuming a pest-free yield and price. Table 12 reports the monetary value of the event MON 863 yield benefit relative to no corn rootworm control reported in Table 9 for a variety of prices, yields, and untreated root ratings. With moderate corn rootworm pressure (an untreated root rating of 3), the average value of corn rootworm control with event MON 863 is around $50/ac for many farmers. Because of the linear relationship between the untreated root rating and the yield benefit, with low corn rootworm pressure (an untreated root rating of 2), the value is about $25/ac and with high corn rootworm pressure (an untreated root rating of 4), the value is about $75/ac. These values do not include the cost of a technology fee for use of event MON 863. Also, the wide confidence interval indicates the tremendous uncertainty in the actual value realized by a farmer. Because the uncertainty in the realized yield benefit for event MON 863 is similar in magnitude to the uncertainty in the yield benefit for a soil insecticide, farmers will likely discount the average value of event MON 863 for this uncertainty at a level similar to what they currently discount the average value of a soil insecticide.

Table 13 reports the value of the event MON 863 yield benefit relative to applying a soil insecticide for a variety of prices, yields, and untreated root ratings. Event MON 863 has greater value because it provides slightly better control. With moderate corn rootworm pressure (an untreated root rating of 3), event MON 863 on average provides a yield benefit worth about $8/ac. With low corn rootworm pressure (an untreated root rating of 2), the value is about $4/ac and with high corn rootworm pressure (an untreated root rating of 4), the value is about $12/ac. However, the confidence interval indicates that the value of the additional realized yield benefit likely ranges between $0-$40/ac, depending on the price, yield, and untreated root rating.

Summary

Data from Gray and Steffey (1998) and event MON 863 field trials were used to estimate the yield benefit for hybrids containing event MON 863 relative to corn without corn rootworm control and relative to corn receiving a soil insecticide. Over typical ranges for corn rootworm population pressure, event MON 863 provides a yield benefit of 9-28% relative to no control and of 1.5-4.5% relative to control with a soil insecticide.

For a reasonable range of prices and yields, the value of the event MON 863 yield benefit is \$25-\$75/ac relative to no control and \$4-\$12/ac relative to control with a soil insecticide, depending on corn rootworm pressure.

The field data used to develop these estimates indicate that a tremendous amount of variability in the yield benefit of event MON 863 exists. This uncertainty is not a property of event MON 863, but the of corn-insect agroecosystem in which many environmental and agronomic factors interact to determine a corn plant's yield and yield response to root damage. As a result, the 95% confidence intervals around the average yield benefits are quite wide. At an aggregate level of analysis, such as at the county, state or regional level, the estimated yield benefits remain the same, but the confidence intervals would be narrower. However, at the field level, these confidence intervals indicate the uncertainty a farmer faces concerning the realized yield benefit on a particular field for a particular year. As a result, farmers will discount these average values to account for this uncertainty.

These results require several qualifications. First, these values only refer to the yield benefit. No attempt is made to evaluate the other benefits of event MON 863. In addition, this analysis does not include any cost differences for the different corn rootworm control methods. Such adjustments are fairly easy to incorporate, since an event MON 863 technology fee is non-random. Furthermore, the analysis does not quantify the discount that farmers are likely to use to adjust the average yield benefit of event MON 863 to account for the uncertainty in the actual yield benefit realized.

The analysis also makes biological assumptions that may not be valid. The model was developed using data from soil insecticide experiments, then applied to event MON

863. This model assumes corn rootworm damage and the associated yield loss as measured by the root rating are equivalent for corn receiving a soil insecticide and corn containing event MON 863. Corn rootworm larval feeding behavior on the roots of hybrids containing event MON 863 may differ from corn treated with a soil insecticide, which could imply a different yield response by the corn plant. Thus, though the root ratings may be the same for corn receiving a soil insecticide and for corn containing event MON 863, the average yield benefit may differ. The model ignores this possibility, since no data were available. Once field trials are conducted with event MON 863 in elite hybrids and yield data are collected, this possibility can be investigated.

Similarly, this analysis ignores the possibility that hybrids containing event MON 863 provides more consistent control of corn rootworm larvae than soil insecticides. The model assumes that the yield benefit of event MON 863 is as uncertain as the yield benefit of a soil insecticide. However, soil insecticides are applied long before larval hatch, so that the realized level of efficacy depends on several random environmental factors. Hybrids containing event MON 863 should give more consistent control than a soil insecticide, since event MON 863 is a plant-incorporated insecticide, the insecticide is expressed in the root tissues at high concentrations during the period of larval feeding. Again, once field trials are conducted with event MON 863 in elite hybrids and yield data are collected, the potential for hybrids containing event MON 863 to provide more consistent control than soil insecticides can be investigated.

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Table 1. Summary of root rating difference data from Gray and Steffey (1998).

Location-Year	Average	Standard Deviation	Minimum	Maximum	n
DeKalb 1994	1.62	0.66	0.00	3.20	108
DeKalb 1995	0.90	0.62	0.00	2.60	117
DeKalb 1996	1.38	0.66	0.20	3.20	66
Urbana 1994	2.76	0.53	0.80	4.00	115
Urbana 1995	2.68	0.68	0.20	4.00	113
Urbana 1996	2.03	0.52	0.60	3.20	102
1994	2.21	0.82	0.00	4.00	223
1995	1.77	1.10	0.00	4.00	230
1996	1.78	0.66	0.20	3.20	168
DeKalb	1.28	0.71	0.00	3.20	291
Urbana	2.51	0.66	0.20	4.00	330
All sites-years	1.93	0.92	0.00	4.00	621

Table 2. Summary of proportional yield loss data from Gray and Steffey (1998).

Location-Year	Average	Standard Deviation	Minimum	Maximum	n	n < 0
DeKalb 1994	0.033	0.071	-0.210	0.197	108	31
DeKalb 1995	0.197	0.181	-0.436	0.501	117	14
DeKalb 1996	0.059	0.088	-0.112	0.288	66	16
Urbana 1994	0.272	0.157	-0.163	0.808	115	5
Urbana 1995	0.488	0.214	-0.363	0.850	113	2
Urbana 1996	0.197	0.110	-0.123	0.585	102	2
1994	0.156	0.17	-0.210	0.808	223	36
1995	0.340	0.25	-0.436	0.850	230	16
1996	0.143	0.12	-0.123	0.585	168	18
DeKalb	0.105	0.150	-0.436	0.501	291	61
Urbana	0.323	0.207	-0.363	0.850	330	9
All sites-years	0.221	0.212	-0.436	0.850	621	70

Table 3. Efficacy data from event MON 863 field trials.

Year	Location	Source	Average Root Rating by Treatment		
			Control	MON 863	Force
1999	Monmouth, IL	a	3.78	1.70	2.38
1999	William, IA	a	3.95	1.82	2.45
1999	Atlantic, IA	a	4.08	1.93	2.48
1999	Brookings, SD	a	4.35	1.50	2.10
1999	Columbia, MO	a	4.44	1.86	2.55
1999	Tuscola, IL	a	4.98	1.00	2.15
1999	Thomasboro, IL	a	5.03	2.33	2.33
2000	Brownsburg, IN	b	4.04	2.04	2.04
2000	Aurora, NE	b	3.33	1.86	2.52
2000	Elwood, NE	b	3.83	1.92	3.33
2000	Gothemburg, NE	b	4.88	2.17	2.46
2000	North Liberty, IA	b	3.79	2.00	2.42
2000	Oxford, IN	b	3.08	2.00	2.00
2000	West Lafayette, IN	b	4.42	2.17	2.00
2000	Ames, IA	c	2.43	1.18	1.79
2000	Brookings, SD	c	5.23	1.08	2.53
2000	Franklin, IN	c	3.28	1.50	1.46
2000	Jerseyville, IL	c	3.12	1.89	2.32
2000	Monmouth, IL	c	2.54	1.26	1.22
2000	Stanton. MN	c	2.30	1.15	1.60
2000	Stromsburg, NE	c	3.82	1.34	2.59
2000	Thomasboro, IL	c	3.49	1.87	1.93
2000	Ames, IA	d	3.75	1.30	2.15
2000	Blacksburg, VA	d	2.35	2.25	2.00
2000	Clay Center, NE	d	4.35	2.20	2.65
2000	Concord, NE	d	4.05	2.20	2.20
2000	Lafayette, IN	d	2.35	1.08	1.85
2000	Mead, NE	d	2.90	1.25	1.75
2000	North Platte, NE	d	4.60	1.90	2.45
2000	Scottsbluff, NE	d	3.70	1.50	2.65
2000	Yuma, CO	d	4.28	1.80	2.93

[a] Monsanto (2000), Table 5.
[b] Pilcher (2001), Table 1.
[c] Pilcher (2001), Table 2.
[d] Pilcher (2001), Table 3.

Table 4. Estimated parameters (standard errors in parentheses) and goodness of fit measures for various corn rootworm damage functions.

Parameter	Linear	Quadratic	Cobb Douglas	Negative Exponential	Hyperbolic	Sigmoid
α	0.125	0.176	0.176	0.239	0.169	0.241
	(0.00368)	(0.00954)	(0.00789)	(0.00982)	(0.00683)	(0.0174)
β	--	-0.0242	0.346	2.0446	--	-0.0367
		(0.00357)	(0.0501)	(0.548)		(0.00661)
σ	0.015	0.0239	0.139	0.160	0.0222	0.135
	(0.00481)	(0.00911)	(0.0129)	(0.0137)	(0.00803)	(0.0232)
Adjusted R^2*	0.175	0.138	0.115	0.054	0.167	0.112
RMSE	0.193	0.197	0.199	0.206	0.194	0.200

* Since a zero intercept is imposed, the adjusted R^2 is appropriate, as opposed to the R^2.

Table 5. Maximum likelihood parameter estimates for the soil insecticide root rating and the event MON 863 root rating as functions of the untreated root rating.

Parameter	Estimate	Standard Error	p value
β_S	0.359	0.00442	< 0.001
σ_S	0.0713	0.00198	< 0.001
β_M	0.246	0.0336	< 0.001
σ_M	0.105	0.0155	< 0.001

Table 6. Average root ratings for untreated plots (with trap crops planted the previous season) for the Gray and Steffey (1998) data.

	------- Average Untreated Root Rating -------		
Year	DeKalb	Urbana	Both
1994	3.84	5.19	4.54
1995	2.84	4.88	3.84
1996	3.40	4.24	3.91
All	3.34	4.79	4.11

Table 7. Average root ratings for untreated plots (with trap crops planted the previous season) for Iowa State University Department of Entomology (1998, 1999) data.

	------- Average Untreated Root Rating -------		
Location	1998	1999	Both
Ames	5.0	4.9	5.0
Bryant	4.6	--	--
Cedar Rapids	4.1	5.2	4.7
Crawfordsville	3.9	4.0	4.0
Nashua	3.0	4.4	3.7
Sutherland	2.2	3.6	2.9
All Locations	3.8	4.4	4.1

Table 8. Average root ratings for untreated first-year fields in east-central Illinois from O'Neil et al. (2000), Table 1.

Year	Number of Fields	Average Untreated Root Rating
1996	14	2.25
1997	17	3.40
1998	15	2.82
1999	28	2.26
	Average	2.68

Table 9. Estimated average yield benefit for corn hybrids containing event MON 863 relative to no corn rootworm control.

------ Root Ratings ------			Average		
Untreated	MON 863	Change	Yield Benefit	Confidence Interval	
R_N	R_M	$\Delta R_{N,M}$	$\lambda_{N,M}$	Lower	Upper
1	1.00	0.00	0.0%	0.0%	0.0%
2	1.25	0.75	9.4%	0.3%	31.9%
3	1.49	1.51	18.8%	0.6%	57.5%
4	1.74	2.26	28.3%	1.0%	76.6%
5	1.98	3.02	37.7%	1.5%	89.2%
6	2.23	3.77	47.1%	2.2%	96.2%

Table 10. Estimated average yield benefit for control with a soil insecticide relative to no corn rootworm control.

------ Root Ratings ------			Average		
Untreated	Soil Insecticide	Change	Yield Benefit	Confidence Interval	
R_N	R_S	$\Delta R_{N,S}$	$\lambda_{N,S}$	Lower	Upper
1	1.00	0.00	0.0%	0.0%	0.0%
2	1.36	0.64	8.0%	0.2%	27.4%
3	1.72	1.28	16.0%	0.5%	50.5%
4	2.08	1.92	24.0%	0.8%	68.8%
5	2.44	2.56	32.0%	1.2%	82.4%
6	2.80	3.20	40.0%	1.7%	91.5%

Table 11. Estimated average yield benefit for corn hybrids containing event MON 863 relative to control with a soil insecticide.

-------------- Root Ratings --------------				Average		
Untreated	Soil Insecticide	MON 863	Change	Yield Benefit	Confidence Interval	
R_N	R_S	R_M	$\Delta R_{S,M}$	$\lambda_{S,M}$	Lower	Upper
1	1.00	1.00	0.00	0.0%	0.0%	0.0%
2	1.36	1.25	0.11	1.4%	0.0%	5.2%
3	1.72	1.49	0.23	2.8%	0.1%	10.2%
4	2.08	1.74	0.34	4.2%	0.1%	15.1%
5	2.44	1.98	0.45	5.7%	0.2%	19.9%
6	2.80	2.23	0.57	7.1%	0.2%	24.5%

Table 12. Average value of the yield benefit for corn hybrids containing event MON 863 relative to no corn rootworm control, for a range of average yields and prices.

Average Yield	Average Price	Untreated Root Rating	Average Value ($/ac)	Confidence Interval Lower	Upper
160	2.00	2	$ 30.14	$ 0.84	$ 101.93
160	2.00	3	$ 60.27	$ 1.87	$ 184.05
160	2.00	4	$ 90.41	$ 3.17	$ 245.13
160	2.15	2	$ 32.40	$ 0.90	$ 109.58
160	2.15	3	$ 64.79	$ 2.02	$ 197.85
160	2.15	4	$ 97.19	$ 3.41	$ 263.52
160	2.30	2	$ 34.66	$ 0.97	$ 117.22
160	2.30	3	$ 69.31	$ 2.16	$ 211.66
160	2.30	4	$ 103.97	$ 3.65	$ 281.90
140	2.00	2	$ 26.37	$ 0.74	$ 89.19
140	2.00	3	$ 52.74	$ 1.64	$ 161.04
140	2.00	4	$ 79.11	$ 2.78	$ 214.49
140	2.15	2	$ 28.35	$ 0.79	$ 95.88
140	2.15	3	$ 56.69	$ 1.76	$ 173.12
140	2.15	4	$ 85.04	$ 2.99	$ 230.58
140	2.30	2	$ 30.32	$ 0.85	$ 102.57
140	2.30	3	$ 60.65	$ 1.89	$ 185.20
140	2.30	4	$ 90.97	$ 3.19	$ 246.66
120	2.00	2	$ 22.60	$ 0.63	$ 76.45
120	2.00	3	$ 45.20	$ 1.41	$ 138.04
120	2.00	4	$ 67.81	$ 2.38	$ 183.85
120	2.15	2	$ 24.30	$ 0.68	$ 82.18
120	2.15	3	$ 48.59	$ 1.51	$ 148.39
120	2.15	4	$ 72.89	$ 2.56	$ 197.64
120	2.30	2	$ 25.99	$ 0.73	$ 87.92
120	2.30	3	$ 51.98	$ 1.62	$ 158.74
120	2.30	4	$ 77.98	$ 2.74	$ 211.43
100	2.00	2	$ 18.83	$ 0.53	$ 63.71
100	2.00	3	$ 37.67	$ 1.17	$ 115.03
100	2.00	4	$ 56.50	$ 1.98	$ 153.21
100	2.15	2	$ 20.25	$ 0.57	$ 68.49
100	2.15	3	$ 40.50	$ 1.26	$ 123.66
100	2.15	4	$ 60.74	$ 2.13	$ 164.70
100	2.30	2	$ 21.66	$ 0.60	$ 73.26
100	2.30	3	$ 43.32	$ 1.35	$ 132.29
100	2.30	4	$ 64.98	$ 2.28	$ 176.19

Table 13. Average value of the yield benefit for corn hybrids containing event MON 863 relative to control with a soil insecticide, for a range of average yields and prices.

Average Yield	Average Price	Untreated Root Rating	Value ($/ac)	Confidence Interval Lower	Confidence Interval Upper
160	2.00	2	$ 4.53	$0.12	$16.52
160	2.00	3	$ 9.06	$0.24	$32.63
160	2.00	4	$13.60	$0.36	$48.32
160	2.15	2	$ 4.87	$0.13	$17.76
160	2.15	3	$ 9.74	$0.25	$35.07
160	2.15	4	$14.62	$0.39	$51.95
160	2.30	2	$ 5.21	$0.13	$19.00
160	2.30	3	$10.42	$0.27	$37.52
160	2.30	4	$15.64	$0.41	$55.57
140	2.00	2	$ 3.97	$0.10	$14.45
140	2.00	3	$ 7.93	$0.21	$28.55
140	2.00	4	$11.90	$0.31	$42.28
140	2.15	2	$ 4.26	$0.11	$15.54
140	2.15	3	$ 8.53	$0.22	$30.69
140	2.15	4	$12.79	$0.34	$45.45
140	2.30	2	$ 4.56	$0.12	$16.62
140	2.30	3	$ 9.12	$0.24	$32.83
140	2.30	4	$13.68	$0.36	$48.62
120	2.00	2	$ 3.40	$0.09	$12.39
120	2.00	3	$ 6.80	$0.18	$24.47
120	2.00	4	$10.20	$0.27	$36.24
120	2.15	2	$ 3.65	$0.09	$13.32
120	2.15	3	$ 7.31	$0.19	$26.31
120	2.15	4	$10.96	$0.29	$38.96
120	2.30	2	$ 3.91	$0.10	$14.25
120	2.30	3	$ 7.82	$0.20	$28.14
120	2.30	4	$11.73	$0.31	$41.68
100	2.00	2	$ 2.83	$0.07	$10.32
100	2.00	3	$ 5.67	$0.15	$20.39
100	2.00	4	$ 8.50	$0.22	$30.20
100	2.15	2	$ 3.05	$0.08	$11.10
100	2.15	3	$ 6.09	$0.16	$21.92
100	2.15	4	$ 9.14	$0.24	$32.47
100	2.30	2	$ 3.26	$0.08	$11.87
100	2.30	3	$ 6.52	$0.17	$23.45
100	2.30	4	$ 9.77	$0.26	$34.73

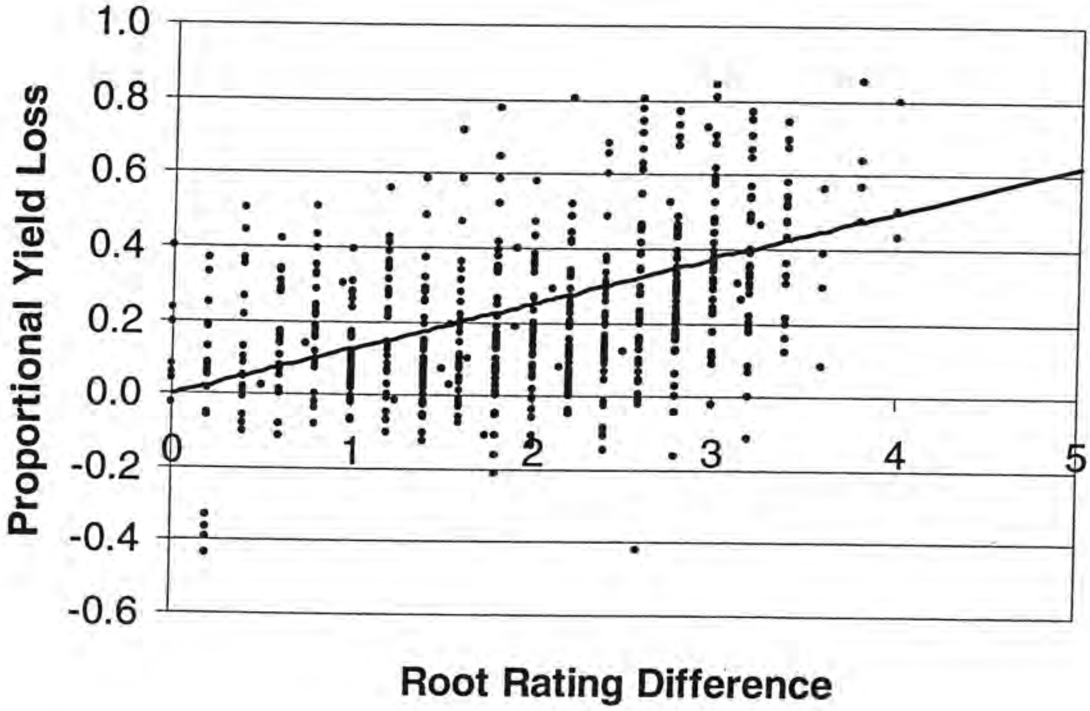

Figure 1. Observed proportional yield loss and predicted mean as a function of the root rating difference.

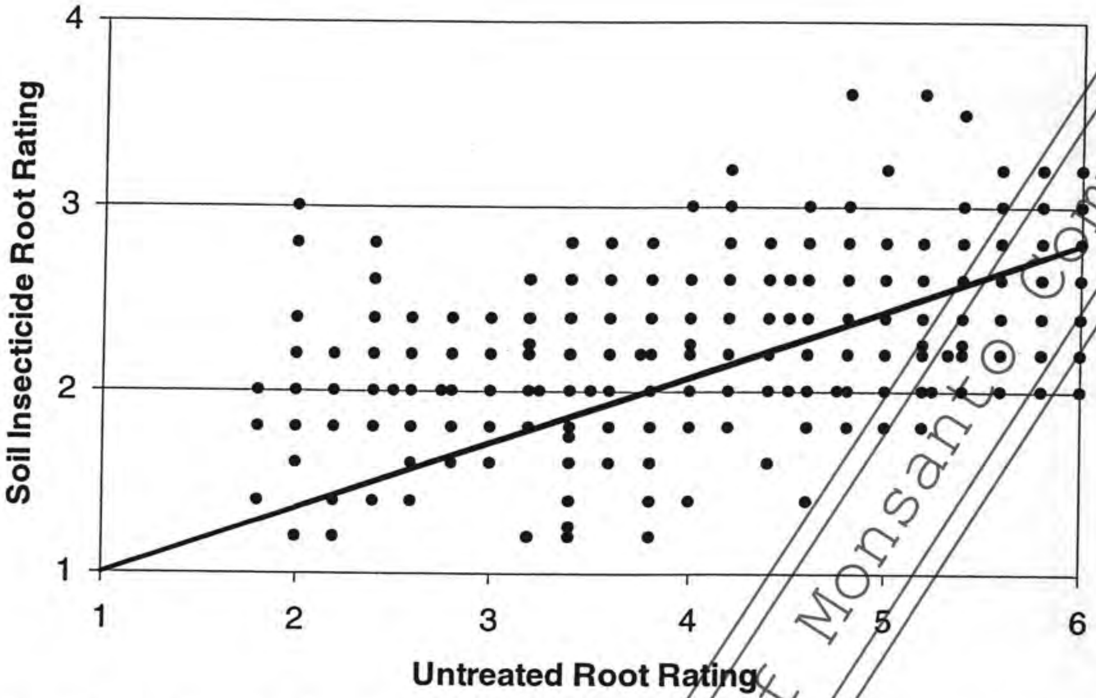

Figure 2. Observed and predicted soil insecticide root rating as a function of the untreated root rating using Gray and Steffey (1998) data.

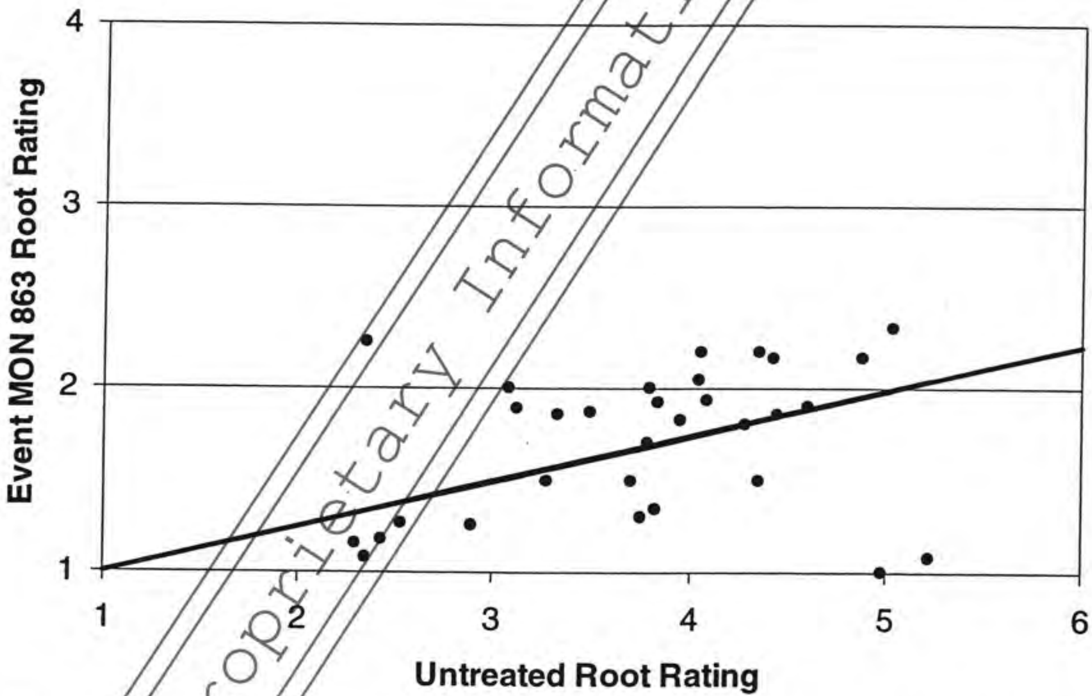

Figure 3. Observed and predicted event MON 863 root rating as a function of the untreated root rating using Monsanto (2000) and Pilcher (2001) data.

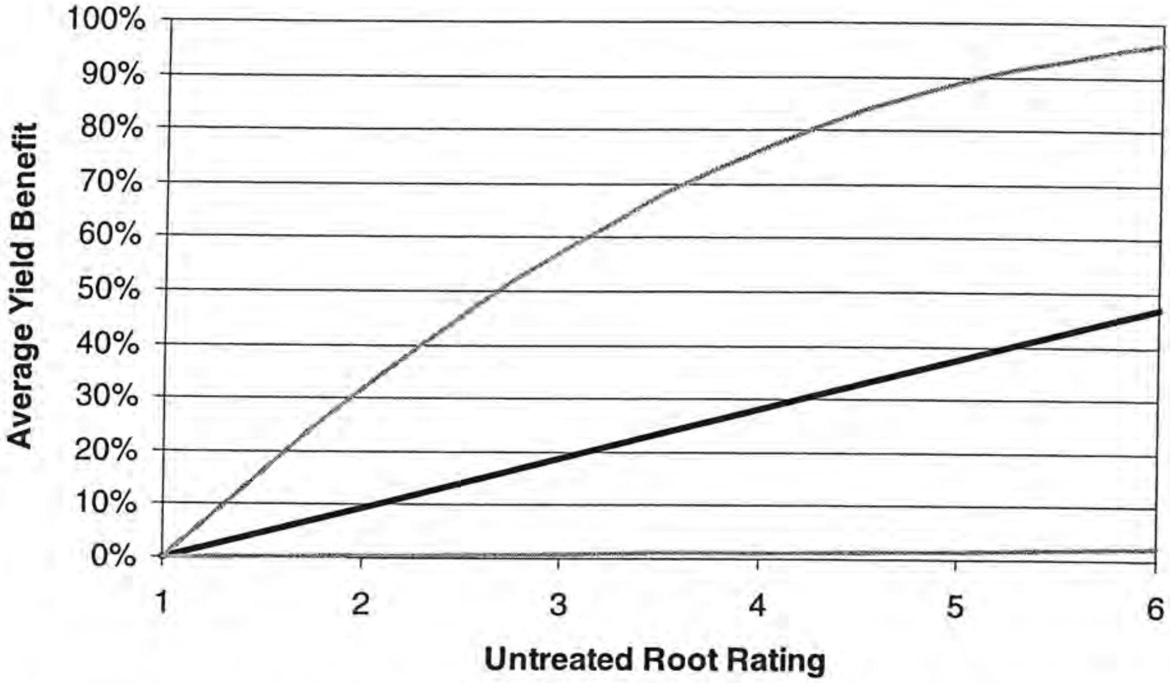

Figure 4. Average yield benefit for hybrids containing event MON 863 relative to no corn rootworm control (black) and the 95% confidence interval (gray).

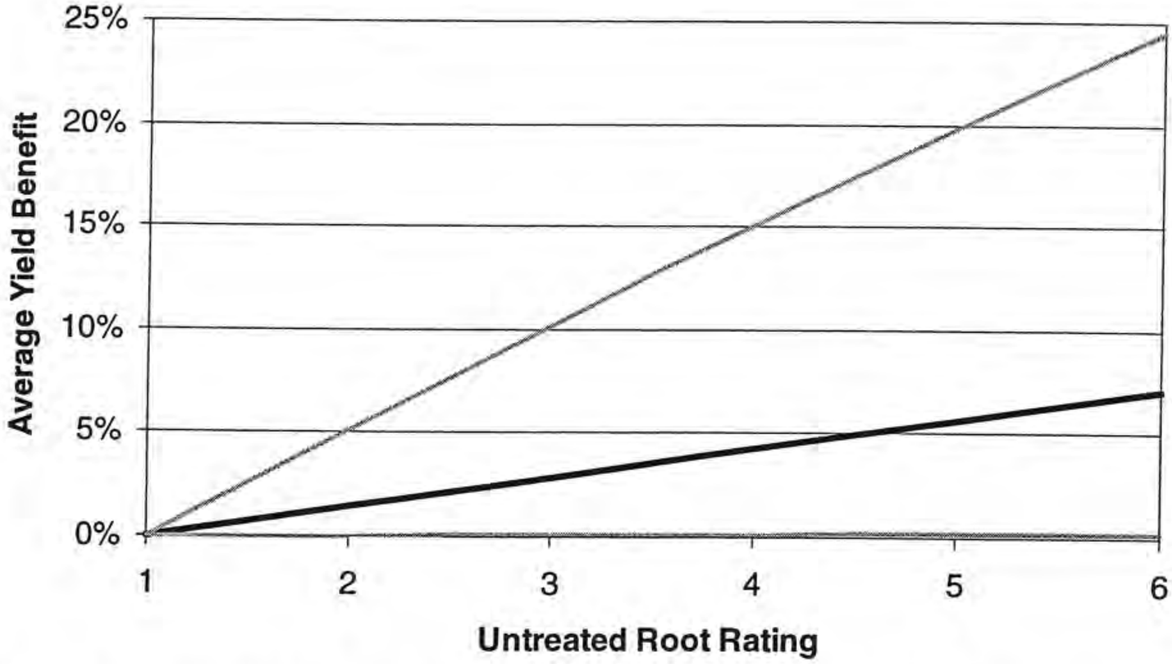

Figure 5. Average yield benefit for hybrids containing event MON 863 relative to control with a soil insecticide (black) and the 95% confidence interval (gray).

APPENDIX A

Composed Error Model for Insect Damage Functions: Yield Impact of Rotation Resistant Western Corn Rootworm in Illinois

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Composed Error Model for Insect Damage Functions and Rotation Resistant Western Corn Rootworm in Illinois

Abstract

This paper describes a composed error model for estimating the conditional distribution of yield loss to serve as an insect damage function. The two-part error separates yield variability due to pest damage from other non-pest factors such as soil heterogeneity, non-uniform application of agronomic practices, and measurement errors. Various common functional forms (linear, quadratic, Cobb-Douglas, negative exponential, hyperbolic, sigmoid) for the pest damage function are presented and parameter estimation is described.

As an empirical illustration, the model is used to estimate a damage function for corn rootworm, the most damaging insect pest of corn in the United States. The estimated damage function gives expected proportional yield loss as a function of the root rating difference and is used to estimate yield loss due to rotation resistant western corn rootworm in east-central Illinois. The estimated average yield loss is 11.6%, more than enough to cover the cost of a soil insecticide application. However, tremendous variability in actual loss exists, so that the probability that actual loss is less than the cost of a soil insecticide ranges 32-45%, depending on the assumed yield and price. As a result, IPM methods potentially have great value, since they can eliminate uneconomical soil insecticide applications.

Keywords: integrated pest management, Monte Carlo integration, root rating, rotation resistance, soil insecticide.

A common problem when using experimental plot data to estimate insect damage functions is the occurrence of "negative losses." For example, a field experiment evaluating a new insecticide may find that the average crop yield for the treated plots exceeds the average yield on the untreated control plots, and that the yield difference is statistically significant. However, for some replicates within the same block, the yield for the untreated control plot exceeds the yield for the plot treated with an insecticide. If the pest is truly damaging, then the yield on a treated plot should always exceed the yield on an untreated plot under equivalent conditions. However, fields are not homogenous. Soil characteristics vary, tillage and nutrient applications are not uniform, yields are measured with error, and the experimental treatments are applied with error. Randomization and replication are used to prevent systematic biases so that valid statistical inferences can be made from the collected data. Assuming proper experimental design, the usual method of analysis is to conduct ANOVA to determine if the difference in mean yields for the treated and untreated plots is statistically significant.

For many types of economic analysis, determining whether a pest control treatment generates a statistically significant yield increase is insufficient. For example, the magnitude of the increase and how it varies with measurable factors such as pest populations is needed for determining an action threshold for integrated pest management (IPM). Similarly, the variance of the yield impact of a pest control treatment quantifies the consistency of the treatment and the risk associated with its use. In these cases, the distribution of the yield difference conditional on observable factors such as the pest population or damage measures is needed. The conditional mean of this distribution can serve as an insect damage function, while the variance of the conditional distribution can be used for analyzing the risk associated with the insect pest or its control.

Unfortunately, yield variability due to soil heterogeneity, the non-uniformity of tillage, nutrient and pesticide applications, and yield measurement error is confounded with yield variability due to the treatment. As a result, assuming all the observed yield variability is due to the pest or the treatment effect over estimates the impact of the pest or the treatment on yield variability. What is needed is a statistical technique that separately identifies the effect of the pest or the treatment on yield and the effect of these other non-pest factors on yield.

This paper presents a model to separately estimate yield variability due to these two sources. The model is for data from replicated plot experiments that use randomized complete block with split plot treatments to evaluate a pest control treatment such as an insecticide, the most common experimental design for such evaluations. The estimated conditional distribution for yield loss is an insect damage function and characterizes the yield risk due to insect damage. The model uses a composed error that separates observed yield variability into two components: (1) a mean-zero normal error to capture yield variability due to soil heterogeneity, non-uniformity of agronomic practices, measurement errors, and similar factors and (2) a strictly positive error to capture yield variability due to pest damage. Characteristics of various useful forms of the model are presented and parameter estimation is described. As an empirical illustration, the composed error model is used to estimate a damage function for corn rootworm, a group of related insect species that are the most damaging insect pest of corn in the United States. Using data from the experiments of Gray and Steffey, the estimated damage function is used to estimate expected yield loss due to rotation resistant western corn rootworm in east-central Illinois.

Composed Error Model

Let Y_c and Y_t respectively denote the measured yield on the control plot and the treated plot. Each treated plot receives a pest control treatment such as an insecticide that reduces or eliminates the pest population. The control plot paired with each treated plot receives no pest control treatment and so should suffer more pest damage than the treated plot. Define λ as proportional yield loss due to pest damage:

(1) $$\lambda = (Y_t - Y_c)/Y_t .$$

If the control yield exceeds the treated yield, λ is negative, while the opposite is true if the treated yield exceeds the control yield.

The composed error model uses two independent errors, a normal (Gaussian) error ε and a strictly positive error δ. Specifically, the model assumes

(2) $$\lambda = 1 - \exp(-(\delta + \varepsilon)).$$

The normal error ε has a zero mean and variance σ^2, while δ has an exponential distribution with mean θ. This specification ensures that $-\infty < \lambda < 1$, which is the range consistent with the definition of λ in equation (1). Maximum loss occurs if $Y_c = 0$ and $Y_t > 0$, when equation (1) gives $\lambda = 1$. The other extreme occurs if the treated plot completely fails and $Y_c > 0$ and $Y_t = 0$, when equation (1) gives $\lambda = -\infty$.

For notation, define $y = \delta + \varepsilon$. Meeusen and van Den Broeck report the probability density function for a random variable such as y. From their specification, the appendix derives an alternative expression for $g(y)$, the probability density function of y:

(3) $$g(y)=\frac{1}{\theta}\exp\left(\frac{\sigma^2-2\theta y}{2\theta^2}\right)\left[1-\Phi\left(\frac{\sigma^2-\theta y}{\sigma\theta}\right)\right],$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Given $g(y)$ and using the transformation of variable technique, the density function for $\lambda = 1-\exp(-y)$ is:

(4) $$h(\lambda)=\frac{1}{\theta}(1-\lambda)^{\frac{1-\theta}{\theta}}\exp\left(\frac{\sigma^2}{2\theta^2}\right)\left[1-\Phi\left(\frac{\sigma^2+\theta\ln(1-\lambda)}{\sigma\theta}\right)\right]$$

for $\lambda < 1$, and 0 otherwise (see appendix).

The mean of λ is $\mu_\lambda = \frac{1+\theta-\exp(0.5\sigma^2)}{1+\theta}$ and the variance is $\sigma_\lambda^2 = \frac{\exp(2\sigma^2)}{1+2\theta} - \frac{\exp(\sigma^2)}{(1+\theta)^2}$ (see appendix). Figure 1 plots the probability density function $h(\lambda)$ for a variety of parameter values to illustrate the ability of the composed error model to capture a wide range of shapes for the distribution of proportional losses, from relatively symmetric to highly skewed in either direction. Maximum likelihood can be used to estimate θ and σ and their standard errors, then the results used to test whether the expected proportional yield gain due to the treatment is zero ($\mu_\lambda = 0$).

Conditional Models

Often the goal of field experiments is not to determine whether a treatment has a significant yield effect. For example, to develop an IPM action threshold, pest populations are measured in order to predict when yield loss will be sufficient to justify the expense of a pest control. Alternatively, the goal may be to assess damage *ex post* in order to determine when economic yield loss has occurred. Both cases require estimating

μ_λ as a function of an observed x, where x is some measure of the pest population or a damage assessment signal.

Maximum likelihood is useful for estimating this type of conditional model. For models presented here, assume that the treatment does not affect σ, the variance parameter of the normal error. Estimation first requires specifying a function $\theta = q(x)$ to describe how θ depends on x, then substituting this $\theta = q(x)$ into equation (4) to obtain the likelihood function in terms of x. Maximum likelihood can then be used to estimate σ and the parameters of $q(x)$. In the empirical section, several commonly desired functional forms for the damage function are derived and estimated to illustrate.

Purged Models

After estimating σ and θ, or a conditional model with $\theta = q(x)$, it is often desirable to drop the variability in yield loss due to the ε error and focus solely on variability due to the pest. For example, if the goal is to determine yield variance due to a specific pest, or the impact of pest control on yield variance. We term these "purged models," since yield loss has been purged of any dependence on non-pest factors captured by the ε error and only depends on the pest effect as captured by the δ error. To differentiate between the full and purged models, use $\tilde{\lambda}$ for proportional yield loss with the purged model, where $\tilde{\lambda} = 1 - \exp(-\delta)$.

The purged model first requires estimating the full model $\lambda = 1 - \exp(-(\delta + \varepsilon))$, then setting $\sigma = 0$ to purge the variability in yield loss of dependence on the non-pest factors associated with the ε error and using only the estimated θ or parameters of $\theta = q(x)$. The probability density function for $\tilde{\lambda}$ is

$$(5) \qquad h(\tilde{\lambda}) = \left((1-\tilde{\lambda})^{\frac{1-\theta}{\theta}} \right) / \theta$$

for $0 \le \tilde{\lambda} \le 1$, and 0 otherwise (see appendix). The cumulative distribution function is

$$(6) \qquad H(\tilde{\lambda}) = 1 - (1-\tilde{\lambda})^{1/\theta}.$$

Note that $\theta = q(x)$ in both equations if applicable. Once purged of non-pest factors, pest damage logically cannot exceed 100%, or be negative, a range consistent with the range $0 \le \tilde{\lambda} \le 1$ imposed by the purged model. The mean and variance are $\mu_{\tilde{\lambda}} = \frac{\theta}{1+\theta}$ and $\sigma_{\tilde{\lambda}}^2 = \frac{\theta^2}{(1+\theta)^2(1+2\theta)}$ (see appendix). Figure 2 plots the probability density function for a variety of parameter values to illustrate the range of possible shapes.

Corn Rootworm Damage Function

To illustrate the composed error model, a conditional distribution for proportional yield loss is estimated and used as a corn rootworm pest damage function. Corn rootworm is a complex of related species that is the most damaging insect pest of corn in the United States. Yield losses and control costs have been estimated to exceed $1 billion annually (Metcalf). Generally, the most problematic species in the complex are the western corn rootworm *(Diabrotica virgifera virgifera)* and the northern corn rootworm (*Diabrotica barberi*), but in some areas the southern corn rootworm (*Diabrotica undecimpunctata howardi*) and the Mexican corn rootworm (*Diabrotica virgifera zeae*) are more damaging. Corn rootworm adult females lay eggs in the summer. These eggs hatch the next spring and the larvae feed on the roots of corn plants. These larvae pupate and emerge as adults from the soil in late summer, then mate and lay eggs.

Larval feeding damage results in direct yield loss and makes corn plants more likely to lodge and suffer additional yield loss. Because corn rootworm larvae feed almost exclusively on corn roots, females generally lay eggs in existing corn fields. As a result, crop rotations with a single year of corn are a widely used control strategy, since eggs laid in a corn field during the summer will hatch in field planted to a non-corn crop the next spring. For continuous or multi-year corn rotations, soil insecticides applied at plant are the most common control strategy in the central and eastern Corn Belt.

In recent years, yield losses and control costs have been increasing because of the development and spread of rotation resistance among western corn rootworm (Levine and Oloumi-Sadeghi). Adult females of rotation resistant western corn rootworm lay eggs not only in corn, but also in other crops. As a result, in areas where a corn-soybean rotation is common, eggs laid in soybean fields hatch in a corn field the next spring. The emerging adults mate and increase the genes responsible for this alternative egg-laying behavior among the population. Rotation resistance first appeared in east-central Illinois and has spread eastward into Indiana, Michigan and Ohio (Onstad et al.).

Because hundreds of corn rootworm larvae can infest a single plant and root feeding occurs underground, accurately measuring larval populations is difficult. As a result, corn rootworm larval damage is usually assessed by a root rating, after larvae have pupated and emerged as adults. The root rating is a measure of corn root damage based on the number of corn root nodes exhibiting feeding scars or completely destroyed by corn rootworm larval feeding. Though other root rating scales exist, the most widely used is the 1 to 6 scale of Hills and Peters, in which 1 indicates no corn rootworm feeding damage and 6 indicates three or more root nodes completely destroyed.

The composed error model is applied to estimate a corn rootworm damage function for use in estimating the annual expected yield loss due to corn rootworm in first-year corn in east-central Illinois, where rotation resistance originated. Data from experiments comparing yields and root ratings for plots treated with soil insecticide and untreated control plots are used for the estimation. The probability density function for proportional yield loss is estimated conditional on the difference in root ratings between the soil-insecticide treated and untreated plots. Field data collected in east-central Illinois concerning root ratings in untreated first-year corn are then used to determine the unconditional distribution of the root rating difference and thus the expected proportional yield loss due to rotation resistant western corn rootworm.

Conditional Distribution of Proportional Yield Loss

Three years (1994-1996) of data from experiments conducted in near Urbana, Illinois were used for estimation (Gray and Steffey). Whole plot treatments were 12 commonly grown hybrids. Sub-plot treatments were 2 rows treated with the soil insecticide Counter® (terbufos) and 2 untreated rows. Depending on the year and location, 8-10 replicates for each hybrid were planted. Collected data included machine-harvested yield and the average root rating for five plants, using the 1-6 scale of Hills and Peters. Only data with treated and untreated yields and root ratings for both sub-plots were used, so that both the root rating difference and proportional yield loss could be calculated. The final result was 330 observations of the soil insecticide yield (Y_t) and average root rating (R_t) and the untreated control yield (Y_c) and average root rating (R_c). Proportional yield loss λ is calculated via equation (1) and the root rating difference x is calculated as $x = R_c - R_t$. Table 1 summarizes the data used for estimation.

Several common functional forms for expected proportional loss conditional on the root rating difference were estimated, i.e. $E[\lambda \mid x] = \mu_\lambda(x)$. A zero intercept was imposed so that plots with equal root ratings have the same expected yield. Note that a zero intercept may not be desired for all applications. Table 2 reports the required functions $\theta = q(x)$ for several functional forms for $\mu_{\bar{\lambda}}(x)$. For notation, $\omega = \exp(0.5\sigma^2)$ and α and β be parameters to estimate. For the purged model, $\sigma = 0$, so that $\omega = 1$. Model names in Table 2 describe the functional form of $\mu_{\bar{\lambda}}(x)$ for the purged model, not the conditional mean $\mu_\lambda(x)$ of the full model or of $q(x)$.

Table 3 reports maximum likelihood parameter estimates and standard errors, as well as goodness of fit and model selection measures, for each model. The adjusted R^2 and root mean square error (RMSE) were calculated using $\mu_\lambda(x)$, the conditional mean of the full model, since the data were fit to this mean. The adjusted R^2 and RMSE support the linear model, while the Likelihood Dominance Criterion (Pollack and Wales) and Akaike's Information Criterion (AIC) support the Cobb-Douglas model. Given these mixed results, we selected the linear model since it is both parsimonious in terms of the number of parameters and gives the best fit. Figure 3 illustrates the fit and indicates why the adjusted R^2 and RMSE are low for all models. The data show tremendous variation in proportional yield loss for the same root rating difference, so that no univariate model can provide a good fit.

The conditional mean of the purged model, $\mu_{\bar{\lambda}}(x)$, is appropriate for a corn rootworm damage function, since only yield variability due to corn rootworm is pertinent. As a result, proportional yield loss follows the probability density reported in equation

(5), where $\theta = q(x)$ as reported in Table 2, with parameters as reported in Table 3. Thus mean proportional yield loss is $\mu_{\bar{\lambda}}(x) = 0.114x$ for the linear model. The cumulative distribution given by equation (6) allows calculation of a 95% confidence interval around the purged model's predicted mean.

Empirical Application

Conditional Distribution of the Root Rating Difference

The Gray and Steffey data were used to estimate the probability density function for the root rating difference conditional on the untreated root rating. The root rating difference has upper and lower limits. When the untreated root rating is 6 and the soil insecticide treated root rating is 1, the root rating difference reaches its maximum of 5. The minimum of zero occurs when the two root ratings are equal, assuming that the untreated root rating must equal or exceed the treated root rating. The minimum and maximum in the data are 0.2 and 4.0 respectively.

Given the existence of upper and lower limits, a conditional beta distribution is assumed. Plots indicated a linear or quadratic relationship between the mean root rating difference and the untreated root rating with a constant standard deviation. A zero intercept was imposed, so that no root rating difference is expected when the untreated root rating indicates no corn rootworm damage. The lower and upper limits of the distribution were fixed at 0.0 and 5.0. Specific models for the linear and quadratic means are $\mu_x(R_c) = r_1(R_c - 1)$ and $\mu_x(R_c) = r_1(R_c - 1) + r_2(R_c - 1)^2$, with constant standard deviation σ_x.

The standard beta density with parameters ν and γ has mean $\mu_x = \nu / (\nu + \gamma)$ and variance $\sigma_x^2 = \nu\gamma / [(\nu+\gamma)^2(\nu+\gamma+1)]$. Solving these equations for ν and γ gives $\nu = [\mu_x^2(1-\mu_x)/\sigma_x^2] - \mu_x$ and $\gamma = [\mu_x(1-\mu_x^2)/\sigma_x^2] - (1-\mu_x)$. Substituting the linear or quadratic equation for μ_x into these gives the density function in terms of σ_x, r_1, and r_2 so that maximum likelihood can be used to estimate these parameters. Table 4 reports parameter estimates and standard errors for both models. Because all reported goodness of fit and model selection measures support the quadratic model, the quadratic model is used for this analysis. Figure 4 illustrates the fit.

Unconditional Distribution of the Untreated Root Rating

The experiments conducted by Gray and Steffey used late-planted corn the previous season as a trap crop to ensure high corn rootworm larval populations. As a result, their data for the untreated root rating are not indicative of the unconditional distribution of the untreated root rating in first-year corn. O'Neil et al. report monitoring data from first-year corn fields of several cooperating farmers in different counties in east-central Illinois for 1996-1999. Collected data included the average and standard deviation of the root rating in several untreated fields. These root rating data indicate the natural pressure from rotation resistant western corn rootworm laying eggs in soybeans.

Table 5, adapted from O'Neil et al. Table 1, reports the mean and standard deviation of the untreated root rating each season. Because a root rating must range 1 to 6, the beta density is an appropriate choice for the unconditional distribution for the untreated root rating in first-year corn. First, the reported means and standard deviations are rescaled to the standard beta density range of 0 to 1. For the mean, rescaling requires subtracting the minimum of 1 and dividing by the range of 6 – 1 = 5, and for the standard deviation, rescaling requires dividing by the range of 5. Table 5 reports the ν and γ for the standard beta density consistent with the rescaled means and standard deviations for each year, using the equations for ν and γ as functions of the mean μ and variance σ^2. For notation, denote the implied rescaled untreated root rating as $\tilde{R}_c = (R_c - 1)/5$.

Assuming that the rescaled untreated root rating follows a beta density with a ν and γ equal to the average ν and γ reported in Table 5 would underestimate its actual variability. As a result, a hierarchical model is specified, in which the parameters ν and γ follow a bivariate normal distribution with means and variance -covariance matrix as reported in Table 5.

Empirical Results

For the specified model, the unconditional expected value of proportional yield loss is $E[\lambda] = \alpha E[x]$ and $E[x] = 5.0(r_1(E[R_c] - 1) + r_2(E[R_c^2] - 2E[R_c] + 1))$. However, calculating $E[R_c] = E[\nu/(\nu+\gamma)]$ and $E[R_c^2]$, where ν and γ follow a bivariate normal distribution, is analytically intractable. As a result, Monte Carlo integration (Greene p. 192-195) is used to estimate $E[R_c]$ and $E[R_c^2]$. Similarly, the unconditional variance of proportional yield loss is $Var[\lambda] = \alpha^2 Var[x]$. However, the unconditional $Var[x]$ is not the σ_x^2 reported in Table 4, since σ_x was estimated conditional on R_c. As a result, Monte Carlo methods are also used to estimate the unconditional $Var[x]$ and obtain a 95% confidence interval for λ.

A C++ program using algorithms reported in Press et al. and Cheng drew random variables from the bivariate normal and beta distributions. First 5,000 draws of ν and γ

from the bivariate normal distribution were obtained, then for each pair, 5,000 draws of the scaled untreated root rating $\tilde{R}_c$ from the beta distribution were obtained, for a total of 25 million draws. Each $\tilde{R}_c$ was then transformed to R_c by multiplying by 5 and adding 1. The average of these R_c and the squared R_c is the Monte Carlo integral estimate of E[R_c] and E[R_c^2] respectively.

To estimate Var[λ] and obtain a 95% confidence interval required further Monte Carlo draws of the root rating difference x and proportional yield loss λ. Each R_c was used to parameterize the beta density and draw a root rating difference x. By inverting the cumulative distribution for the purged model and using the Inverse Transform Method (Cheng), a random draw for λ is $\lambda = 1-(1-u)^{\theta}$, where u is a uniform random variable and $\theta = \alpha x/(1-\alpha x)$. The equation for θ is derived from the reported equation in Table 3 for the linear model, but for the purged form of the model with $\omega = 1$, since $\sigma^2 = 0$. The average of these λ and λ^2 is a Monte Carlo estimate of E[λ] and E[λ^2], so that the Monte Carlo estimate of Var[λ] = E[λ^2] – E[λ]^2. Similarly, the lower 2.5% and upper 97.5% quantiles are Monte Carlo estimates of the 95% confidence interval.

Table 6 reports all Monte Carlo estimates, as well as the correct values for those that can be determined analytically. The unconditional expected proportional yield loss due to rotation resistant corn rootworm in untreated first-year corn in east-central Illinois is 0.116. The standard deviation is 0.125 and the lower and upper limits of the 95% confidence interval are 0.00149 and 0.460 respectively. These results indicate that not only is yield loss on average is quite substantial, but also quite variable.

Converting these proportional yield loss estimates into revenue loss requires using an expected yield and price and assuming that corn rootworm damage is independent of yield and price. Table 7 reports the expected revenue loss, as well as the lower and upper limits of the 95% confidence interval, using parameter estimates in Table 6. Estimates of the direct cost of purchasing and applying a soil insecticide for corn rootworm control typically range \$12-\$15/ac. Thus, the estimated revenue loss is on average more than enough to cover the direct cost of a soil insecticide.

The tremendous variability in the actual yield loss realized implies that though on average the direct cost will be covered, the probability that the cost will not be covered in a specific year on a specific field is substantial. The last column in Table 7 reports Monte Carlo estimates of these probabilities for the different yield and price assumptions. The revenue loss for each Monte Carlo draw of λ was calculated, then the losses were sorted and the cumulative probability for each loss determined empirically. Table 7 reports the probabilities that revenue loss < \$15/ac.

In general the average losses in Table 7 indicate that farmers should be concerned about corn rootworm damage on first-year corn in east-central Illinois. However, the probabilities that the loss is less than \$15/ac in Table 7 are large and indicate that applying a soil insecticide on all first-year corn acres will quite often result in a revenue loss, since the cost of the soil insecticide will not be recovered. As a result, an IPM method that measures the adult corn rootworm population or egg laying in soybean fields to be planted in corn the next season could be profitable if scouting costs are low and provide reliable information. As an example of such an IPM method, O'Neil et al. have developed an economic threshold using Pherocon AM traps to measure adult populations in soybean fields.

Conclusion

This paper describes a composed error model for use with experimental plot data to estimate a conditional distribution for yield loss to serve as an insect damage function. The model uses a two-part error to separate yield variability due to pest damage from other factors such as soil heterogeneity, non-uniform application of agronomic practices, and measurement errors. Various functional forms for the pest damage function are presented for the conditional model and parameter estimation is described.

As an empirical illustration, the composed error model is used to estimate a damage function for corn rootworm, the most damaging insect pest of corn in the United States. Using data from the experiments of Gray and Steffey, the estimated damage function is used to estimate expected yield loss due to rotation resistant western corn rootworm in east-central Illinois. The estimated average yield loss is 11.6%, which is more than enough to cover the cost of a soil insecticide application which typically ranges \$12-\$15/ac. However, tremendous variability in actual loss exists, so that the probability that actual loss is less than \$15/ac ranges 32-45%, depending on the assumed yield and price. As a result, IPM practices such as described by O'Neil et al. potentially have value, since they can eliminate uneconomical soil insecticide applications.

Various improvements or extensions of the composed error model are possible. The conditional models reported in Table 2 impose a zero intercept, so that the insect pest can only cause non-negative damage. However, some experimental evidence indicates that at low populations, some insect pests can actually increase yields by stimulating plant growth. Similarly, the zero-intercept form of the composed error model prevents estimating any negative impacts that pest control may have, such as crop damage due to herbicide application or a "yield drag" due to a transgenic gene conferring herbicide or insect resistance. As a result, some applications require models without a zero-intercept.

Additionally, the composed error model specified here uses an exponential error to capture yield variability due to the pest. The exponential error is quite restrictive in terms of the shape of the probability density function and has only one parameter. As a result, estimating models with flexible relationships for both the conditional mean and conditional variance is difficult. More flexible conditional models require a different error assumption for the pest effect, but deriving the associated composed error for the joint distribution of the errors can become difficult.

Table 1. Summary statistics for proportional yield loss and root rating difference data from Gray and Steffey used for estimation.

Variable	Year	Average	Standard Deviation	Minimum	Maximum	n	n < 0
Proportional Yield Loss	1994	0.272	0.157	-0.163	0.808	115	5
	1995	0.488	0.214	-0.363	0.850	113	2
	1996	0.197	0.110	-0.123	0.585	102	2
	Pooled	0.323	0.207	-0.363	0.850	330	9
Root Rating Difference	1994	2.76	0.53	0.8	4.0	115	0
	1995	2.68	0.68	0.2	4.0	113	0
	1996	2.03	0.52	0.6	3.2	102	0
	Pooled	2.51	0.67	0.2	4.0	330	0

Table 2. Required functions $\theta = q(x)$ for the full model that give common functional forms for the conditional mean of proportional yield loss for the purged model.

Functional Form	Purged Model $\mu_{\tilde{\lambda}}(x)$	Full Model $\mu_{\lambda}(x)$	Required $\theta = q(x)$
Linear	αx	$\alpha x \omega$	$\dfrac{\omega - 1 + \alpha x \omega}{1 - \alpha x \omega}$
Quadratic	$\alpha x + \beta x^2$	$\alpha x \omega + \beta x^2 \omega$	$\dfrac{\omega - 1 + \alpha x \omega + \beta x^2 \omega}{1 - \alpha x \omega - \beta x^2 \omega}$
Cobb-Douglas	αx^{β}	$\alpha x^{\beta} \omega$	$\dfrac{\omega - 1 + \alpha x^{\beta} \omega}{1 - \alpha x^{\beta} \omega}$
Negative Exponential	$\alpha(1 - \exp(-\beta x))$	$\alpha(1 - \exp(-\beta x))\omega$	$\dfrac{\omega - 1 + \alpha(1 - \exp(-\beta x))\omega}{1 - \alpha(1 - \exp(-\beta x))\omega}$
Hyperbolic	$\dfrac{\alpha x}{\alpha x + 1}$	$\dfrac{\alpha x}{\alpha x + \omega}$	$\omega - 1 + \alpha x$
Sigmoid	$\dfrac{\alpha x + \beta x^2}{\alpha x + \beta x^2 + 1}$	$\dfrac{\alpha x + \beta x^2}{\alpha x + \beta x^2 + \omega}$	$\omega - 1 + \alpha x + \beta x^2$

Table 3. Estimated parameters (standard errors in parentheses) and goodness of fit measures for various corn rootworm damage functions.

Parameter	Linear	Quadratic	Cobb Douglas	Negative Exponential	Hyperbolic	Sigmoid
α	0.114	0.191	0.218	0.311	0.177	0.291
	(0.00398)	(0.0155)	(0.0238)	(0.0237)	(0.00987)	(0.0400)
β	--	-0.0297	0.286	1.0369	--	-0.0413
		(0.00541)	(0.115)	(0.350)		(0.0128)
σ	0.237	0.343	0.357	0.353	0.293	0.344
	(0.0481)	(0.0634)	(0.0674)	(0.0653)	(0.0503)	(0.0613)
Adjusted R^2*	0.123	0.060	0.060	0.053	0.109	0.062
RMSE	0.194	0.200	0.200	0.201	0.195	0.200
Log-likelihood	116.0	128.3	131.1	130.9	124.7	130.0
AIC	-228.0	-250.5	-256.2	-255.7	-245.3	-253.9

* Since a zero intercept is imposed, the adjusted R^2 is appropriate (Greene p. 255).

Table 4. Estimated parameters (standard errors in parentheses) and goodness of fit measures for the linear and quadratic mean models of the distribution of the root rating difference conditional on the untreated root rating.

Parameter	Linear	Quadratic
r_1	0.667	0.544
	(0.00491)	(0.0309)
r_2	--	0.0304
		(0.00758)
σ_x	0.348	0.336
	(0.0131)	(0.0127)
Adjusted R^2*	0.732	0.744
RMSE	0.342	0.335
Log-likelihood	-113.9	-106.0
AIC	231.7	218.1

* Since a zero intercept is imposed, the adjusted R^2 is appropriate (Greene p. 255).

Table 5. Data concerning the unconditional distribution of the untreated root rating in first-year corn in east-central Illinois.

		------- Reported* -------		----- Rescaled -----			
Year	n	Mean	Standard Deviation	Mean	Standard Deviation	ν	γ
1996	14	2.25	0.16	0.250	0.032	45.53	136.58
1997	17	3.40	0.19	0.480	0.038	82.49	89.36
1998	15	2.82	0.20	0.364	0.040	52.30	91.39
1999	28	2.26	0.15	0.252	0.030	52.53	155.91
					Average	58.21	118.31
					Variance	204.4	827.6
					Covariance	−247.3	

* Source: O'Neil et al., p. 100, Table 1.

Table 6. Monte Carlo estimates of various statistics concerning yield loss due to rotation resistant western corn rootworm in east-central Illinois.

Statistic	Monte Carlo Estimate	Correct Value
$E[\nu]$	58.21	58.21
$E[\omega]$	118.31	118.31
$Var[\nu]$	272.35	272.52
$Var[\omega]$	1103.1	1103.5
$Cov[\nu,\omega]$	-246.7	-247.3
$Cor[\nu,\omega]$	-0.450	-0.451
$E[R_c]$	2.694	---
$E[R_c^2]$	7.610	---
$E[x]$	1.020	---
$E[\lambda]$	0.116	---
$Var[\lambda]$	0.0156	---
Standard Deviation of λ	0.125	---
2.5% Quantile of λ	0.00149	---
97.5% Quantile of λ	0.460	---

Table 7. Monte Carlo estimated expected revenue loss due to rotation resistant western corn rootworm in first-year corn in east-central Illinois for a variety of yield and price assumptions, as well as the probability that the revenue loss is < $15/ac.

Yield (bu/ac)	Price ($/bu)	Expected Revenue Loss ($/ac)	95% Confidence Interval Lower ($/ac)	95% Confidence Interval Upper ($/ac)	Probability Revenue Loss < $15.00
120	2.00	27.84	0.36	110.30	0.448
120	2.15	29.93	0.38	118.58	0.427
120	2.30	32.02	0.41	126.85	0.408
130	2.00	30.16	0.39	119.50	0.425
130	2.15	32.42	0.41	128.46	0.405
130	2.30	34.68	0.44	137.42	0.387
140	2.00	32.48	0.42	128.69	0.404
140	2.15	34.92	0.45	138.34	0.385
140	2.30	37.35	0.48	147.99	0.368
150	2.00	34.80	0.45	137.88	0.386
150	2.15	37.41	0.48	148.22	0.367
150	2.30	40.02	0.51	158.56	0.351
160	2.00	37.12	0.47	147.07	0.369
160	2.15	39.90	0.51	158.10	0.351
160	2.30	42.69	0.55	169.13	0.335
170	2.00	39.44	0.50	156.26	0.355
170	2.15	42.40	0.54	167.98	0.336
170	2.30	45.36	0.58	179.70	0.321

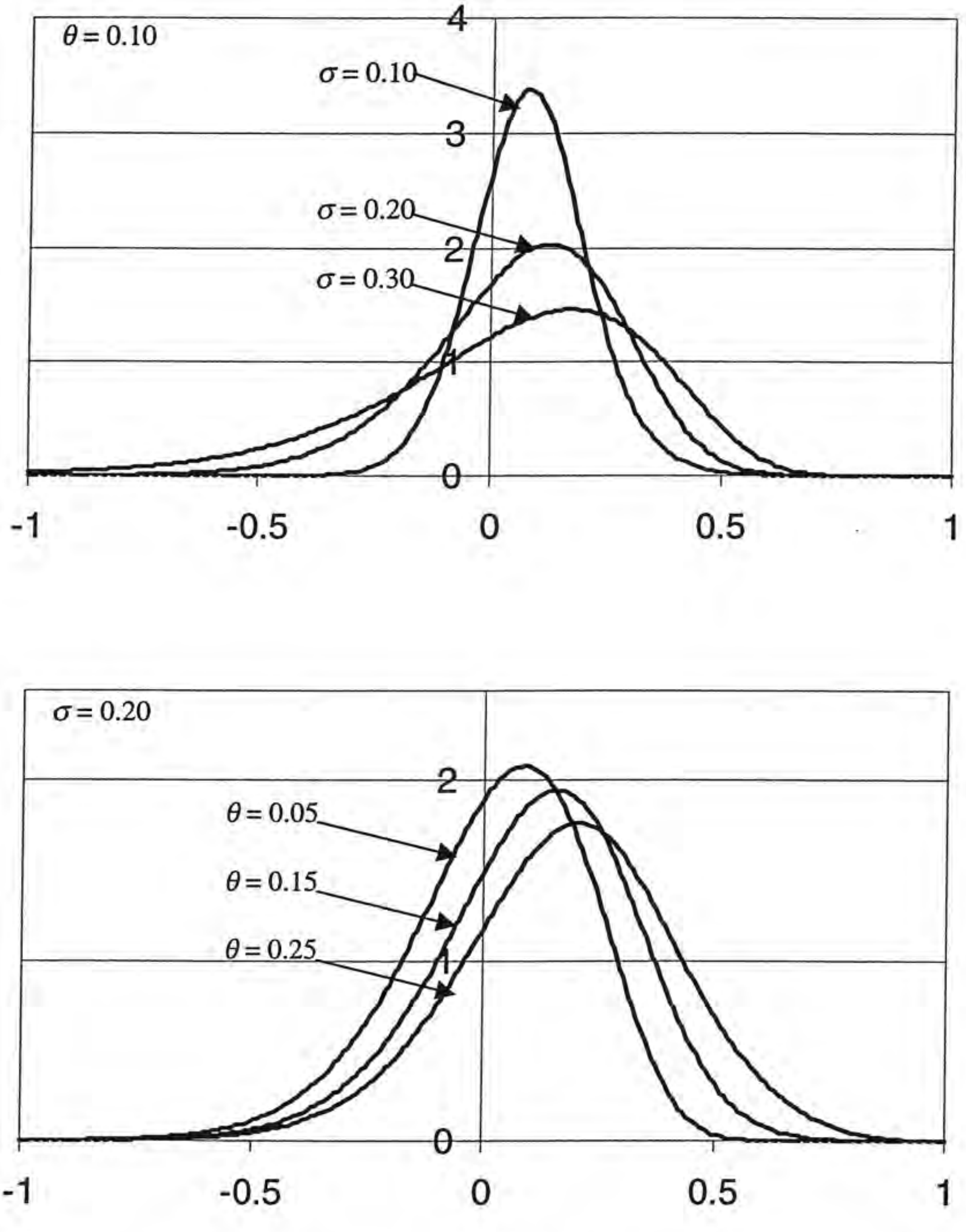

Figure 1. Probability density function $h(\lambda)$ with $\theta = 0.10$ and $\sigma = 0.10$, 0.20, and 0.30 (top) and $\sigma = 0.20$ and $\theta = 0.05$, 0.15, and 0.25 (bottom).

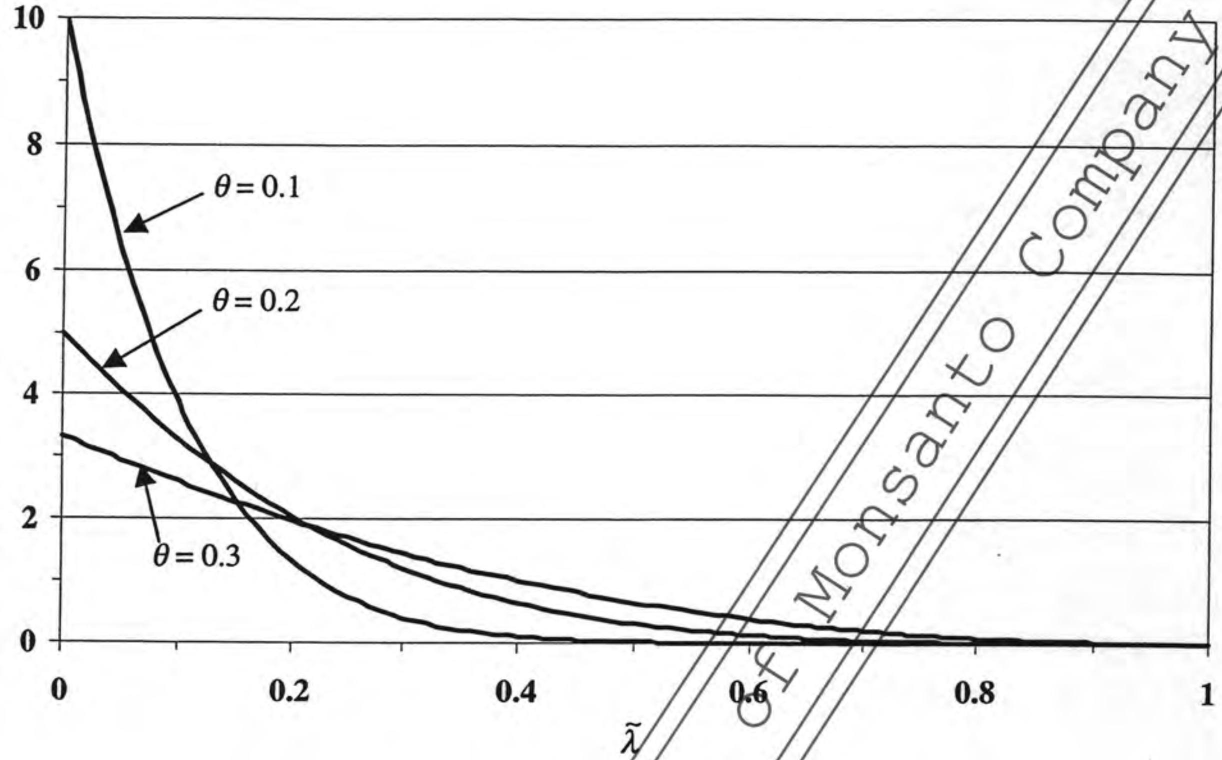

Figure 2. Probability density function $h(\tilde{\lambda})$ with $\theta = 0.10$, 0.20, and 0.30.

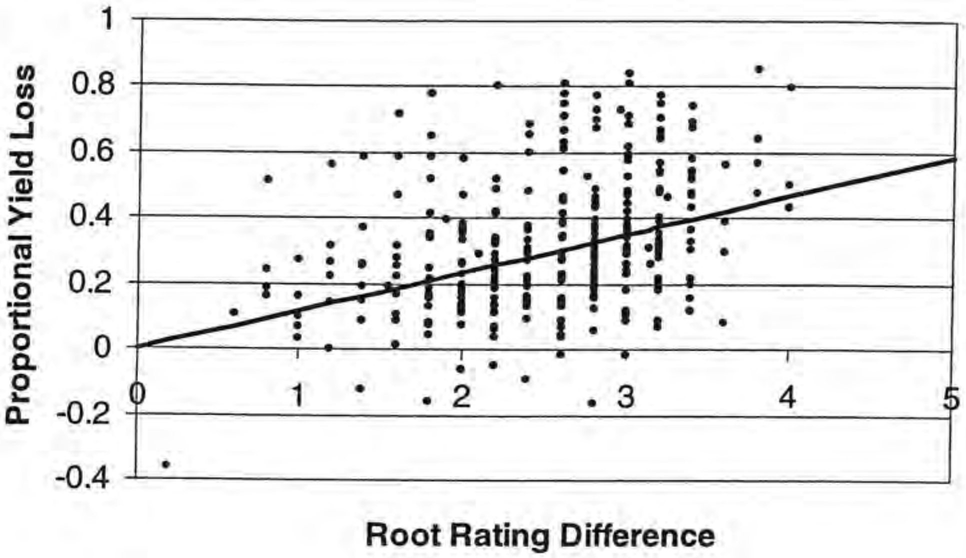

Figure 3. Observed proportional yield loss and predicted mean as a function of the root rating difference for the linear model.

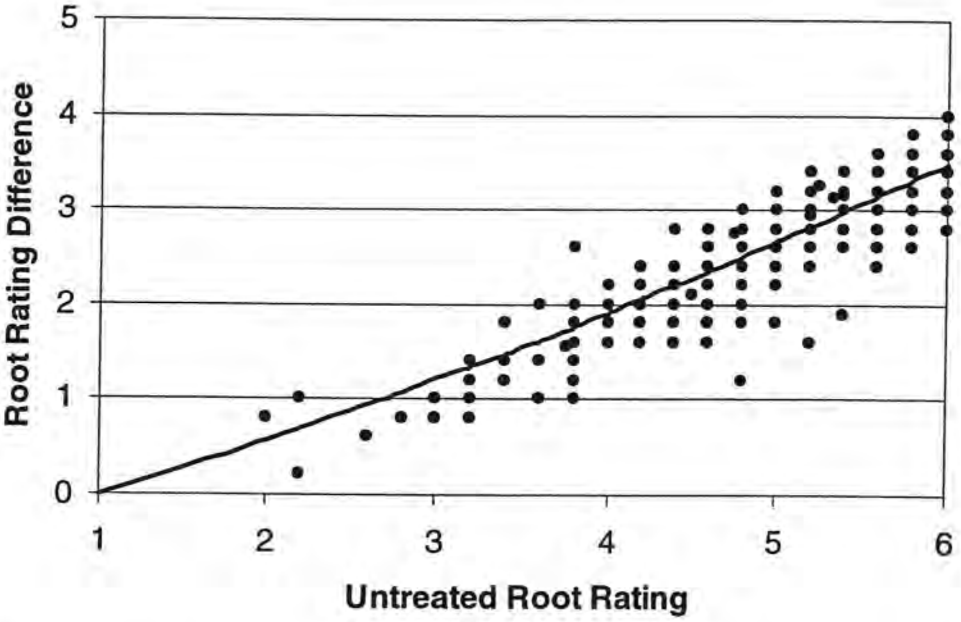

Figure 4. Observed root rating difference and predicted mean as a function of the untreated root rating for the quadratic model.

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Appendix

Derivation of equation (3)

Meeusen and van Den Broeck report the probability density function for $w = z + v$, where z has an exponential distribution with mean $1/\lambda$ and v is normal with zero mean and variance σ^2. Converting their notation to the notation used in this paper, $w = y$, $z = \delta$, $v = \varepsilon$ and the parameters $\theta = 1/\lambda$ and $\sigma^2 = \sigma^2$. Making these conversions, their equation (4) is the probability density function of y:

(A1) $$g(y) = \frac{1}{2\theta}\exp\left(\frac{\sigma^2 - 2\theta y}{2\theta^2}\right)\operatorname{erfc}\left(\frac{\sigma^2 - \theta y}{\sigma\theta\sqrt{2}}\right),$$

where $\operatorname{erfc}(\cdot) = 1 - \operatorname{erf}(\cdot)$ is the complementary error function and $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-s^2)ds$ is the error function (Press et al., p. 220). Greene (p.187) reports that $\Phi(x) = 0.5 + 0.5\operatorname{erf}(x/\sqrt{2})$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Rearrange this expression to obtain $1 - \operatorname{erf}(x/\sqrt{2}) = 2(1 - \Phi(x))$, and then use this result to give $\operatorname{erfc}\left(\frac{\sigma^2 - \theta y}{\sigma\theta\sqrt{2}}\right) = 2\left(1 - \Phi\left(\frac{\sigma^2 - \theta y}{\sigma\theta}\right)\right)$. Substitute this into equation (A1) and simplify to obtain equation (3).

Derivation of equation (4)

Given probability density function $g(y)$ for y, the transformation of variable technique gives the probability density function $h(\lambda)$ for $\lambda = 1 - \exp(-y)$. Since $y = -\ln(1-\lambda)$ and $\partial y / \partial\lambda = \frac{1}{1-\lambda}$:

$$h(\lambda) = g(y(\lambda))\left|\partial y / \partial\lambda\right|$$

$$h(\lambda) = \frac{1}{\theta}\exp\left(\frac{\sigma^2 + 2\theta\ln(1-\lambda)}{2\theta^2}\right)\left[1 - \Phi\left(\frac{\sigma^2 + \theta\ln(1-\lambda)}{\sigma\theta}\right)\right]\left|\frac{1}{1-\lambda}\right|$$

(A2) $$h(\lambda) = \frac{1}{\theta}\exp\left(\frac{\sigma^2}{2\theta^2}\right)\exp\left(\frac{\ln(1-\lambda)}{\theta}\right)\left(\frac{1}{1-\lambda}\right)\left[1 - \Phi\left(\frac{\sigma^2 + \theta\ln(1-\lambda)}{\sigma\theta}\right)\right].$$

Because $\exp\left(\frac{\ln(1-\lambda)}{\theta}\right) = \exp(\ln(1-\lambda))^{\frac{1}{\theta}} = (1-\lambda)^{\frac{1}{\theta}}$, $\exp\left(\frac{\ln(1-\lambda)}{\theta}\right)\left(\frac{1}{1-\lambda}\right)$ simplifies to

$\frac{(1-\lambda)^{\frac{1}{\theta}}}{1-\lambda} = (1-\lambda)^{\frac{1}{\theta}-1} = (1-\lambda)^{\frac{1-\theta}{\theta}}$. Substitute this simplification into equation (A2) to obtain equation (4).

Derivation of mean and variance of λ

By equation (2), $\lambda = 1 - \exp(-\delta)\exp(-\varepsilon)$. Define two random variables $a = \exp(-\delta)$ and $b = \exp(-\varepsilon)$, so that $\lambda = 1 - ab$. As Evans, Hastings and Peacock report, since $\varepsilon \sim N(0, \sigma^2)$, *b* has a lognormal distribution with mean and variance

(A3) $$\mu_b = \exp(0.5\sigma^2)$$

(A4) $$\sigma_b^2 = \exp(2\sigma^2) - \exp(\sigma^2).$$

Since δ has an exponential distribution with mean θ, it has probability density function $w(\delta) = \exp(-\delta/\theta)/\theta$. The transformation of variable technique gives *f*(*a*), the probability density function of *a*:

(A5) $$f(a) = \frac{1}{\theta}a^{\frac{1}{\theta}-1},$$

for $0 \le a \le 1$ and 0 otherwise. The mean of *a* is $\mu_a = \int_0^1 af(a)da = \frac{1}{\theta}\int_0^1 a^{\frac{1}{\theta}}da$, which is

(A6) $$\mu_a = \frac{1}{1+\theta}.$$

The variance of *a* is $\sigma_a^2 = \int_0^1 a^2 f(a)da - \left(\int_0^1 af(a)da\right)^2$. The first term is $\int_0^1 a^2 f(a)da =$

$\frac{1}{\theta}\int_0^1 a^2 a^{\frac{1}{\theta}-1}da = \frac{1}{\theta}\int_0^1 a^{\frac{1}{\theta}+1}da = \frac{1}{1+2\theta}$. Using (A6), the second term is $\frac{1}{(1+\theta)^2}$. Thus

$\sigma_a^2 = \int_0^1 a^2 f(a)da - \left(\int_0^1 af(a)da\right)^2 = \frac{1}{1+2\theta} - \frac{1}{(1+\theta)^2}$, which can be simplified:

(A7) $$\sigma_a^2 = \frac{\theta^2}{(1+2\theta)(1+\theta)^2}.$$

The mean of $\lambda = 1 - ab$ is $\mu_\lambda = 1 - \mu_a\mu_b$ because *a* and *b* are independent, since δ and ε are independent. Using (A3) and (A5),

(A8) $$\mu_\lambda = 1 - \frac{\exp(0.5\sigma^2)}{1+\theta}.$$

The variance of $\lambda = 1 - ab$ is $\sigma_\lambda^2 = \mathrm{Var}[ab]$. Because *a* and *b* are independent,

$\mathrm{Var}[ab] = \sigma_a^2\sigma_b^2 + \sigma_a^2\mu_b^2 + \sigma_b^2\mu_a^2$. Substitute (A3)-(A5) into this equation and simplify:

$$\sigma_\lambda^2 = \frac{\theta^2\left(\exp(2\sigma^2) - \exp(\sigma^2)\right)}{(1+2\theta)(1+\theta)^2} + \frac{\theta^2\exp(\sigma^2)}{(1+2\theta)(1+\theta)^2} + \frac{\exp(2\sigma^2) - \exp(\sigma^2)}{(1+\theta)^2}$$

$$\sigma_\lambda^2 = \frac{\theta^2\exp(2\sigma^2) - \theta^2\exp(\sigma^2) + \theta^2\exp(\sigma^2)}{(1+2\theta)(1+\theta)^2} + \frac{\exp(2\sigma^2) - \exp(\sigma^2)}{(1+\theta)^2}$$

$$\sigma_\lambda^2 = \frac{\theta^2\exp(2\sigma^2) + (1+2\theta)\left(\exp(2\sigma^2) - \exp(\sigma^2)\right)}{(1+2\theta)(1+\theta)^2}$$

$$\sigma_{\lambda}^{2} = \frac{\theta^{2}\exp(2\sigma^{2}) + (1+2\theta)\exp(2\sigma^{2}) - (1+2\theta)\exp(\sigma^{2})}{(1+2\theta)(1+\theta)^{2}}$$

$$\sigma_{\lambda}^{2} = \frac{(1+2\theta+\theta^{2})\exp(2\sigma^{2}) - (1+2\theta)\exp(\sigma^{2})}{(1+2\theta)(1+\theta)^{2}}$$

$$\sigma_{\lambda}^{2} = \frac{(1+\theta)^{2}\exp(2\sigma^{2}) - (1+2\theta)\exp(\sigma^{2})}{(1+2\theta)(1+\theta)^{2}}$$

(A9) $$\sigma_{\lambda}^{2} = \frac{\exp(2\sigma^{2})}{(1+2\theta)} - \frac{\exp(\sigma^{2})}{(1+\theta)^{2}}.$$

Derivation of equation (5)

By definition, $\tilde{\lambda} = 1 - \exp(-\delta) = 1 - a$, where $a = \exp(-\delta)$. From (A8), a has probability density function $f(a) = \frac{1}{\theta} a^{\frac{1}{\theta}-1}$, so that the transformation of variable technique can be used to find $h(\tilde{\lambda})$, the probability distribution function of $\tilde{\lambda}$. Because $\tilde{\lambda} = 1 - a$, $a = 1 - \tilde{\lambda}$ and $\left|\partial a / \partial \tilde{\lambda}\right| = |-1| = 1$. Thus

(A10) $$h(\tilde{\lambda}) = f(a(\tilde{\lambda}))\left|\partial a / \partial \tilde{\lambda}\right| = \frac{1}{\theta}(1-\tilde{\lambda})^{\frac{1}{\theta}-1}.$$

Derivation of mean and variance of $\tilde{\lambda}$

Because $\tilde{\lambda} = 1 - a$, $\mu_{\tilde{\lambda}} = \mathrm{E}[\tilde{\lambda}] = 1 - \mathrm{E}[a]$. By equation (A6), $\mathrm{E}[a] = \frac{1}{1+\theta}$, so that

(A11) $$\mu_{\tilde{\lambda}} = 1 - \frac{1}{1+\theta} = \frac{1+\theta-1}{1+\theta} = \frac{\theta}{1+\theta}.$$

Similarly, $\sigma_{\tilde{\lambda}}^{2} = \mathrm{Var}[\tilde{\lambda}] = \mathrm{Var}[1-a] = \mathrm{Var}[a] = \sigma_{a}^{2}$, which is reported in equation (A7).